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A772

IX. PROCESSING AND TRANSMISSION OF INFORMATION[*]

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A. CORRECTION OF LOW-DENSITY BURSTS

In many communication channels noise disturbances occur in the form of low-density bursts. Codes capable of correcting multiple bursts of low density are efficient tools for error control in such channels. One such class of codes has recently been developed. It makes use of the dual of product codes whose component codes are majority-logic-decodable cyclic codes. The new codes that have been developed are also majority-decodable.

R. T. Chien

B. DOCUMENT STORAGE SCHEME BASED ON POLARIZED DISTANCE

Some mathematical properties of term-matching document retrieval systems have been developed. These properties can be used as a basis for a new file-organization scheme. Some of the advantages of this scheme are that the key-to-address transformation is easily determined, the documentary information is stored only once in a file, the file organization allows the use of various matching functions and thresholds, and the dimensionality of the transform is easily expanded to accommodate data bases of various sizes.

R. T. Chien

C. DIGITAL DATA NETWORKS WITH FINITE STORAGE

In a recently completed Master's thesis,[1] a model was presented of a digital data store-and-forward communication network of arbitrary topology containing a finite

amount of storage at nodes. The problem is to maximize the size of the multicommodity flow, which is the sum of labeled flows between source-receiver pairs. The maximization is carried out under the assumption that always there are available messages awaiting transmission from each source. Routing strategies and storage allocation which maximize throughput are specified.

The theoretical maximum for multicommodity throughput is found as a special case of a theorem that was proved in the thesis. Let I represent some digital data communication network which places integral constraints on flow and permits only finite storage at nodes. Let II be a network of the same topology as network I with the same node and branch capacities, but with no integral constraints on flow and no storage at nodes. The theorem proves that, given any flow over a sufficiently long period of time in I, there exists a flow in II that is greater than, or equal to, the average of the given flow in I. As applied in the thesis, the theorem shows that multicommodity throughput in II is never worse than any average of multicommodity throughput in I, provided a sufficiently long period of time is used to average the throughput. Since a convenient algorithm exists for finding optimal multicommodity throughput in II, the theoretical upper bound on multicommodity throughput is obtained.

It was also found that in general for any given flow in II, there exists a flow in I whose average over a sufficiently long period of time is arbitrarily close to the given flow. Only a finite amount of storage is required at nodes in I. For a given rational flow in II, only a finite amount of storage is needed to permit the average of the flow in I over a sufficiently long period of time to equal the given flow. This result applies to the problem at hand if the given flow in II maximizes the sum of labeled flows between source-receiver pairs. Rational node and branch capacities in II lead to rational optimal multicommodity flow. The routing strategy that was used allows each node to route messages independently of every other node's activities.

J. S. Fields

References

1. J. S. Fields, "Digital Data Networks with Finite Storage," S. M. Thesis, Department of Electrical Engineering, M. I. T., May 12, 1972.

D. INTERACTION FORMULATION OF QUANTUM COMMUNICATION THEORY

1. Interaction between Quantum Systems

Let H_S and H_A be the unperturbed Hamiltonians of the quantum systems S and A, respectively, when they do not interact with each other. Let $\psi^S(t)$ and $\psi^A(t)$ be the wave functions describing the states $|s\rangle$ and $|a\rangle$ of the independent systems S and A. The

wave functions obey the Schrödinger equations

$$i\hbar \frac{\partial \psi^S}{\partial t} = H_S \psi^S(t) \tag{1}$$

$$i\hbar \frac{\partial \psi^A}{\partial t} = H_A \psi^A(t). \tag{2}$$

The state of the combined system S+A can be represented by a state vector $|s\rangle|a\rangle$ in a tensor product Hilbert Space $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_A$, where $|s\rangle|a\rangle$ is the tensor product of the state $|s\rangle$ in the Hilbert Space $\mathcal{H}_S$ for S and $|a\rangle$ in the Hilbert Space $\mathcal{H}_A$ for A. Let $P_{\mathcal{H}_S}$ and $P_{\mathcal{H}_A}$ be the projection operators onto the constituent spaces $\mathcal{H}_S$ and $\mathcal{H}_A$ of $\mathcal{H}$ and $P_{\mathcal{H}_A}|s\rangle|a\rangle = |a\rangle$ $P_{\mathcal{H}_S}|s\rangle|a\rangle = |s\rangle$. Define the inner product in this tensor product space as $(|s'\rangle|a'\rangle, |s\rangle|a\rangle) = \langle s'|s\rangle\langle a'|a\rangle$, where $\langle s'|s\rangle$ and $\langle a'|a\rangle$ are just inner products on the spaces $\mathcal{H}_S$ and $\mathcal{H}_A$.

Note that the joint state of S+A can be represented as $|s\rangle|a\rangle$ if and only if S and A are independent at that instant. If $\{|\phi_i\rangle\}_{i\in I}$ and $|\psi_j\rangle_{j\in J}$ are the orthonormal basis of H_S and H_A, respectively, then the joint state of S+A can be represented in general by

$$|S+a\rangle = \sum_{\substack{i\in I \\ j\in J}} a_{ij}|\phi_i\rangle|\psi_i\rangle, \tag{3}$$

where the inner product of the two states is defined as

$$\langle s'+a'|s+a\rangle = \sum_{\substack{i\in I \\ j\in J}} a_{ij}^{!*}\langle\psi_i|\langle\phi_i| \sum_{\substack{i\in I \\ j\in J}} a_{ij}|\phi_i\rangle|\psi_i\rangle$$

$$= \sum_{\substack{i\in I \\ j\in J}} a_{ij}^{!*} a_{ij}. \tag{4}$$

For normalization of the state vectors

$$\sum_{\substack{i\in I \\ j\in J}} |a_{ij}|^2 = 1. \tag{5}$$

When the two systems S and A are interacting with each other the evolution in time of the wave function is given by an interaction Hamiltonian H_{AS} in the following Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \psi'^{S+A}(t) = (H_A + H_S + H_{AS}) \psi'^{S+A}(t). \tag{6}$$

Since the problem is to describe the effect of H_{AS}, it is convenient to remove the time dependence associated with H_A and H_S from the equation. This is accomplished by the unitary transformation

$$\psi'^{S+A}(t) = \exp[-i(H_A + H_S)t/\hbar] \, \psi^{S+A}(t), \tag{7}$$

which yields

$$i\hbar \frac{\partial}{\partial t} \psi^{S+A}(t) = H_{AS}(t) \, \psi^{S+A}(t), \tag{8}$$

where

$$H_{AS}(t) = \exp[i(H_A + H_S)t/\hbar] H_{AS} \exp[-i(H_A + H_S)t/\hbar]. \tag{9}$$

Formally, the solution to the interaction problem is

$$\psi^{S+A}(t) = U(t, t_o) \, \psi^{S+A}(t_o), \tag{10}$$

where

$$U(t, t_o) = \left\{ \exp -\frac{i}{\hbar} \int_{t'=t_o}^{t} H_{AS}(t') \, dt' \right\} \tag{11}$$

is a unitary transformation. Let U be the corresponding unitary transformation in the abstract tensor product Hilbert Space $\mathcal{H}$, $|s^f + a^f\rangle$ be the final state of the system S+A, and $|s^o\rangle|a^o\rangle$ be the original independent state for S and A. Then

$$|s^f + a^f\rangle = U|s^o\rangle|a^o\rangle. \tag{12}$$

If eventually the two systems stop interacting and become independent again, then

$$|s^f + a^f\rangle = |s^f\rangle|a^f\rangle = U|s^o\rangle|a^o\rangle. \tag{13}$$

Since the transformation is unitary, all norms and inner products are preserved.

2. Sequential Detection of Signals Transmitted by a Quantum System (Binary Pure State)

Suppose we want to transmit a binary signal with a quantum system S that is not corrupted by noise. The system is in state $|s_0\rangle$ when digit zero is sent, and in state $|s_1\rangle$ when the digit one is sent. (Let p_0 and p_1 be the a priori probabilities that the digits zero and one are sent $p_0 + p_1 = 1$.) The task is to observe the system S and decide whether a "zero" or a "one" is sent. The performance of detection is given by the probability of error. Helstrom has solved this problem, for a single observation of the system S that can be characterized by a Hermitian operator.[1] The probability of error obtained for one simple measurement is $\Pr[\epsilon] = \frac{1}{2}\left[1 - \sqrt{1 - 4P_1 P_0 |\langle s_1|s_0\rangle|^2}\right]$.

We try to consider the performance of a sequential detection scheme by bringing an apparatus A to interact with the system S and then performing a measurement on S and then on A, or vice versa. The structure of the second measurement is optimized as a consequence of the <u>outcome</u> of the first measurement.

Suppose we can find an apparatus A that can interact strongly with the system S so that after the interaction different states of system S will induce different states of system A. Suppose the initial state of the apparatus is known to be $|a_0\rangle$, and the final state is $|a_0^f\rangle$ if S is in state $|s_0\rangle$, and $|a_1^f\rangle$ if S is in state $|s_1\rangle$, and $|a_1^f\rangle \neq |a_0^f\rangle$. By the interaction formalism that has been developed, the inner product of the state that describes the system S+A when digit zero is sent and that which describes it when digit one is sent is invariant under any interaction that can be described by an interaction Hamiltonian H_{AS} that is Hermitian. That is,

$$\langle s_0|s_1\rangle = \langle s_0|s_1\rangle\langle a_0|a_0\rangle = \langle s_0^f|s_1^f\rangle\langle a_1^f|a_0^f\rangle,$$

where $|s_0^f\rangle$ and $|s_1^f\rangle$ are final states of S after interaction if a zero or a one is sent. Now suppose $|\langle s_0|s_1\rangle| < |\langle s_0^f|s_0^f\rangle| < 1$ which implies also $|\langle s_0|s_1\rangle| < |\langle a_0^f|a_1^f\rangle| < 1$. (See Fig. IX-1.) We want to observe S first in an optimal way. The process is similar to Helstrom's in that we choose a measurement that is characterized by a Hermitian operator O_s in the Hilbert Space $\mathcal{H}_s$ so that the probability of error $\Pr[\epsilon_s]$ is minimized.

$$\Pr[\epsilon_s] = \frac{1}{2}\left[1 - \sqrt{1 - 4P_1 P_0 |\langle s_0^f|s_1^f\rangle|^2}\right],$$

and the probability of correct detection is

$$\Pr[\epsilon_s] = \frac{1}{2}\left[1 + \sqrt{1 - 4P_1 P_0 |\langle s_1^f|s_0^f\rangle|^2}\right].$$

Suppose the outcome is one. The a priori probabilities p_1, p_0 of apparatus A being
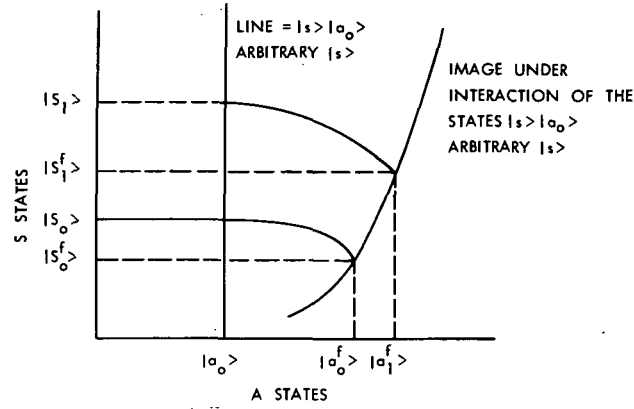
Fig. IX-1. Joint states of system and apparatus before and after interaction.

in states $\left|a_1^f\right\rangle$ and $\left|a_0^f\right\rangle$ has been updated to $\Pr\left[C_s\right]$ and $\Pr\left[\epsilon_s\right]$.

Now we perform a similar second measurement on A, characterized by a Hermitian operator O_A in Hilbert Space $\mathcal{K}_A$. Now we use a new set of a priori probabilities $p_1' = \Pr\left[C_s\right]$, $p_0' = \Pr\left[\epsilon_s\right]$ for the states $\left|a_1^f\right\rangle$ and $\left|a_0^f\right\rangle$. Assuming that we already have all available information from the outcome of the first measurement in the updated a priori probabilities for A, we would base our decision entirely on the second measurement. The optimal Hermitian operator O_A is chosen to minimize the probability of error of detection $\Pr\left[\epsilon\right]$ in a process similar to the first measurement, and the performance is

$$\Pr\left[\epsilon\right] = \frac{1}{2}\left[1 - \sqrt{1 - 4\,\Pr\left[C_s\right]\Pr\left[\epsilon_s\right]\left|\langle a_0^f\,|\,a_1^f\rangle\right|^2}\,\right].$$

But $\Pr\left[C_s\right]\Pr\left[\epsilon_s\right] = P_1 P_0\left|\langle s_1^f\,|\,s_0^f\rangle\right|^2$, and $\langle s_1^f\,|\,s_0^f\rangle\langle a_1^f\,|\,a_0^f\rangle = \langle S_1\,|\,S_0\rangle$, which gives

$$\Pr\left[\epsilon\right] = \frac{1}{2}\left[1 - \sqrt{1 - 4 P_1 P_0\left|\langle S_1\,|\,S_0\rangle\right|^2}\,\right].$$

This is exactly the same performance obtained by Helstrom in one simple operation, so a sequential detection scheme does not offer any improvement. We hope, however, that our analysis will be useful in analyzing detection procedures that involve interaction of carrier field and apparatus. When the single measurement operator cannot be easily realized or implemented, we hope that the sequential method will shed light on the implementation of the optimal detector by breaking up the detection process into processes that are more familiar to experimenters.

V. Chan

References

1. C. W. Helstrom, "Quantum Mechanical Communication Theory," Proc. IEEE 58, 1578-1598 (1970).

E. OPTICAL SCATTER PROPAGATION

For several years, our group, in cooperation with Lincoln Laboratory, M. I. T., has conducted scatter propagation experiments at visible, infrared, and ultraviolet wavelengths in the 10-20 km range. Recently, using resonant discharge lamps, we have begun to explore the characteristics of short-range scatter propagation in the UV ozone absorption band. The configuration is presented and preliminary results of these experiments are reported here.

1. Transmitter

A mercury discharge spectral lamp (Osram Hg_3) was used as the UV source. On the roof of the Green Building, M. I. T., 80 m from the ground, a searchlight of 2-ft diameter with a focal length of 9 11/16 in. was used to collimate the light into a narrow beam. Since the source was cylindrical, the collimated beam pattern was rectangular. The divergent angle was 10^{-2} rad in the horizontal plane and 4×10^{-2} rad in the vertical plane. The source was expected to give out 500 mW in the 2500 Å mercury line. Only ~80 mW was expected to be transmitted in the collimated beam because of a protective glass covering on the tube that shielded some of the light. On a previous line-of-sight photon counting measurement, the total power transmitted in the beam was estimated to be 50 mW.

2. Receiver

The receiver was on the roof of Building 20, M. I. T., 150 m from the Green Building and 15 m above the ground. It was a PMT (RCA 4522) (with a quantum efficiency of 15% at 2500 Å) with a liquid filter and Corning 7-54 filter in front of it. A 5-ft hood in front of the filter assembly limited the field of view to 10^{-1} rad. The total collecting area of the receiver was 20 in.^{2} The transmission characteristics of the whole filter assembly are shown in Fig. IX-2.

2. Analytic Expressions for Received Scattered Power

Figure IX-3 shows the relative transmitter and receiver geometry. Under the assumption of isotropic scattering, the received power expression (Pr) will be different when the whole collimated beam is within the FOV from when it is not.
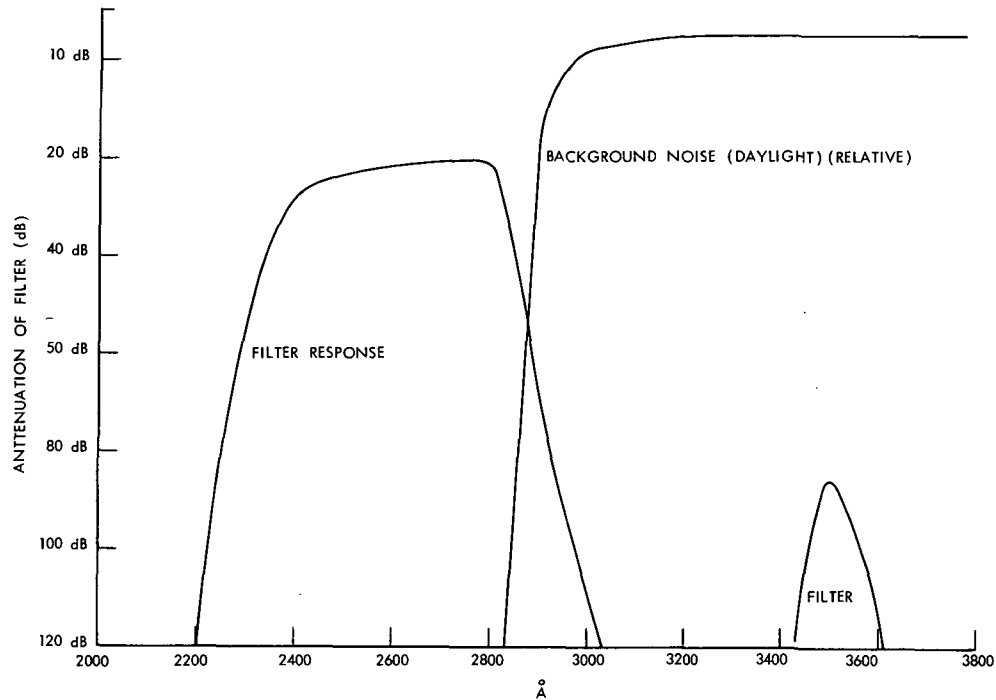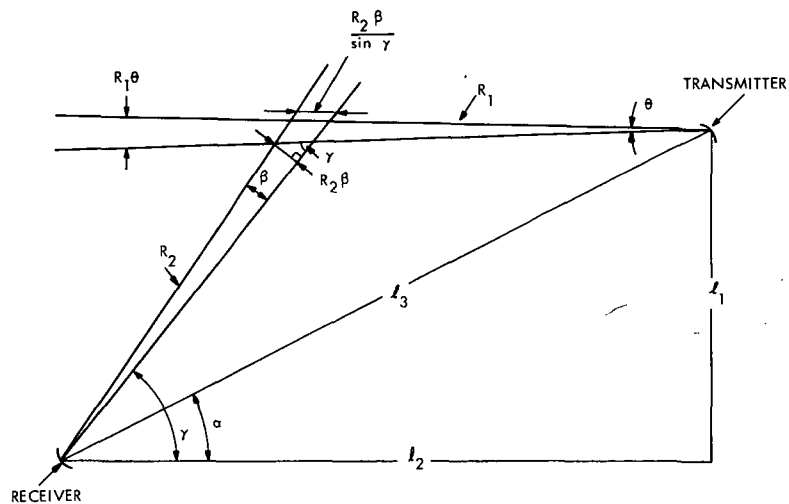
Fig. IX-2. Filter transmission characteristics.

Fig. IX-3. Relative transmitter and receiver geometry.

Case I

$$R_2\beta \geq R_1\theta$$

$$P_r = \frac{Ar\beta\sigma_s P_o}{4\pi\ell_3 \sin a} \exp -(\sigma_a + \sigma_s)\ell_3 \cos a \left(1 + \tan a \tan \frac{\gamma}{2}\right).$$

Case II

$$R_2\beta \leq R_1\theta$$

$$Pr = \frac{Ar\beta\sigma_s P_o}{4\pi\ell_3 \sin (\gamma-a)} \frac{\beta}{\theta} \exp -(\sigma_a + \sigma_s)\ell_3 \cos a \left(1 + \tan a \tan \frac{\gamma}{2}\right).$$

For short paths (<1 km) the exponential factor does not contribute much variation over scattering angles. Cases I and II can be cast in a combined form as
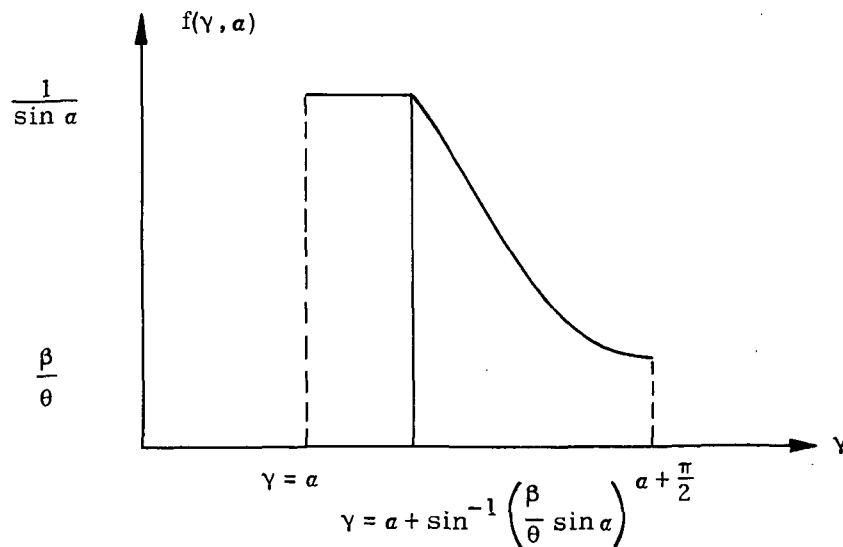
$$Pr = \frac{Ar\beta\sigma_s P_o}{4\pi\ell_3} f(\gamma, a) \exp -(\sigma_a + \sigma_s)\ell_3 \cos a \left(1 + \tan a \tan \frac{\gamma}{2}\right),$$

where

$$f(\gamma, a) = \frac{1}{\sin a} \qquad R_2\beta \geq R_1\theta$$

$$= \frac{\beta}{\theta} \frac{1}{\sin (\gamma-a)} \qquad R_2\beta \leq R_1\theta,$$

which is diagrammed as follows.

When Mei scattering is important the expression for scattered power can be modified by multiplying Pr by a scattering distribution function for the particular wavelength $P(\gamma)$.

4. Photon Counting

A photon counter (Laser Sciences) was used to count pulses from the PMT. The lower threshold setting was at 200 mV and the upper threshold was at 500 mV, and approximately 70% of the pulses were counted. During the photon-counting experiment, because of chemical changes in the liquid filters, the attenuation at 2537 Å was 30 dB instead of 24 dB. Based on a transmitted power of 50 mW the expected number of counts was 15,000 counts/second. In the experiment 4,000-24,000 counts were recorded. Dark-current counts varied from 500 to 2,000 counts.

5. Current Measurement

A PMT was modified for current measurement. The fluid in the liquid filter was changed so that the whole filter assembly had an attenuation of 24 dB at 2537 Å. The number of photoelectrons expected was 60,000/s. The tube had a gain of ~4×10^8, which would mean an expected dc current of 0.4 μA. In the experiment the dc signal current was in the range 1-10 μA. Dark current was in the range 5-10 nA and daytime background noise was 0.3-0.8 μA.

Background-noise current could be reduced by narrowing down the receiver FOV, which was 10^{-1} rad. It was estimated that by narrowing it down to 2×10^{-2} rad, the receiver FOV would still see the entire beam, but auxiliary optics would be required to do so.

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